

# 7-4

## Similarity in Right Triangles



### Vocabulary

#### Review

Underline the correct word to complete the sentence.

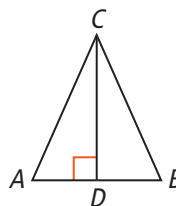
- The *altitude* of a triangle is a segment from a vertex to the opposite side that is parallel / perpendicular to the opposite side.
- In an isosceles triangle, the *altitude* to the base divides the triangle into two congruent / isosceles triangles.
- Circle the *altitude* of  $\triangle ABC$ .

$\overline{AB}$

$\overline{AC}$

$\overline{BC}$

$\overline{CD}$



#### Vocabulary Builder

**geometric mean** (noun) jee uh MEH trik meen

**Definition:** For any two positive numbers  $a$  and  $b$ , the **geometric mean** of  $a$  and  $b$  is the positive number  $x$  such that  $\frac{a}{x} = \frac{x}{b}$ .

**Example:** The **geometric mean** of 4 and 10 is the value of  $x$  in  $\frac{4}{x} = \frac{x}{10}$ , or  $x = 2\sqrt{10}$ .

#### Use Your Vocabulary

- Multiple Choice** Which proportion can you use to find the *geometric mean* of 5 and 15?

(A)  $\frac{x}{5} = \frac{x}{15}$

(B)  $\frac{5}{x} = \frac{15}{x}$

(C)  $\frac{5}{x} = \frac{x}{15}$

(D)  $\frac{5}{15} = \frac{x}{x}$

Underline the correct equation to complete each sentence.

- The *geometric mean*  $x$  of  $a$  and  $b$  is  $x = \sqrt{ab}$  /  $x = ab$ .
- The *geometric mean*  $x$  of 3 and 7 is  $x = \sqrt{21}$  /  $x = 21$ .
- Circle the geometric mean of  $\sqrt{3}$  and  $\sqrt{3}$ .

$\sqrt{3}$

3

$3\sqrt{3}$

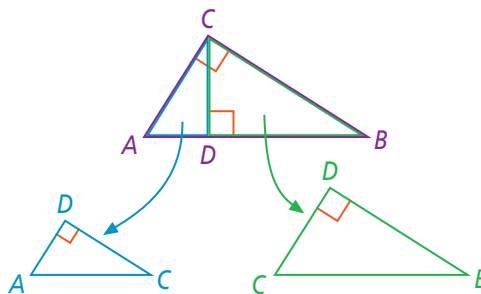
$\sqrt{33}$

# Key Concept Theorem 7-3 and Corollaries 1 and 2

**Theorem 7-3** The altitude to the hypotenuse of a right triangle divides the triangle into two triangles that are similar to the original triangle and to each other.

If ...

$\triangle ABC$  is a right triangle with right  $\angle ACB$ , and  $\overline{CD}$  is the altitude to the hypotenuse



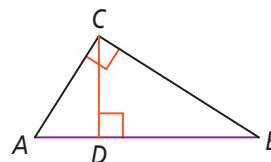
Then ...

$$\begin{aligned}\triangle ABC &\sim \triangle ACD \\ \triangle ABC &\sim \triangle CBD \\ \triangle ACD &\sim \triangle CBD\end{aligned}$$

## Corollary 1 to Theorem 7-3

If ...

The length of the altitude to the hypotenuse of a right triangle is the geometric mean of the lengths of the segments of the hypotenuse.



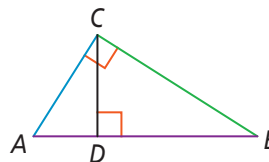
Then ...

$$\frac{AD}{CD} = \frac{CD}{DB}$$

## Corollary 2 to Theorem 7-3

If ...

The altitude to the hypotenuse of a right triangle separates the hypotenuse so that the length of each leg of the triangle is the geometric mean of the length of the hypotenuse and the length of the segment of the hypotenuse adjacent to the leg.



Then ...

$$\frac{AB}{AC} = \frac{AC}{AD}$$

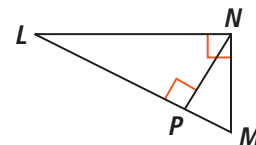
$$\frac{AB}{CB} = \frac{CB}{DB}$$

8.  $\triangle LMN$  is a right triangle with right  $\angle LMN$ .  $\overline{NP}$  is the altitude to the hypotenuse. Complete the similarity statements.

$$\triangle LMN \sim \triangle \text{LNP}$$

$$\triangle LMN \sim \triangle \text{NMP}$$

$$\triangle LNP \sim \triangle \text{NMP}$$



Use the triangle at the right. Write *Corollary 1* or *Corollary 2* for each proportion.

9.  $\frac{c}{a} = \frac{a}{x}$

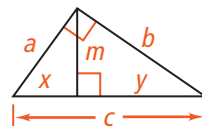
Corollary 2

10.  $\frac{x}{m} = \frac{m}{y}$

Corollary 1

11.  $\frac{c}{b} = \frac{b}{y}$

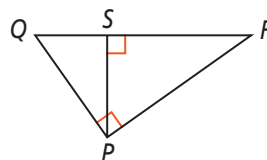
Corollary 2





## Problem 1 Identifying Similar Triangles

**Got It?** What similarity statement can you write relating the three triangles in the diagram?



12. Write the names of the triangles.

$\triangle RPQ$   $\triangle$  **PQS**  $\triangle$  **PRS**

13. Write the three right angles.

$\angle RPQ$   $\angle$  **QSP**  $\angle$  **PSR**

14. Write the three smallest angles.

$\angle QRP$   $\angle$  **SPQ**  $\angle$  **SRP**

15. Use your answers to Exercises 13 and 14 to write three similarity statements beginning with the vertex of the smallest angle in each triangle and ending with the vertex of the right angle.

$\triangle RQP \sim \triangle$  **PQS**

$\triangle RQP \sim \triangle$  **RPS**

$\triangle$  **PQS**  $\sim \triangle$  **RSP**



## Problem 2 Finding the Geometric Mean

**Got It?** What is the geometric mean of 4 and 18?

16. Use the justifications below to find the geometric mean.

$$\frac{4}{x} = \frac{x}{18} \quad \text{Definition of geometric mean}$$

$$x^2 = 72 \quad \text{Cross Products Property}$$

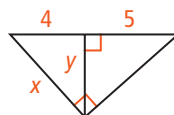
$$x = \sqrt{72} \quad \text{Take the positive square root of each side.}$$

$$x = 6\sqrt{2} \quad \text{Write in simplest radical form.}$$



## Problem 3 Using the Corollaries

**Got It?** What are the values of  $x$  and  $y$ ?



**Underline the correct word to complete each sentence.**

17.  $x$  is the length of a leg of the largest triangle, so use Corollary 1 / Corollary 2 to find the value of  $x$ .

18.  $y$  is the length of the altitude of the largest triangle, so use Corollary 1 / Corollary 2 to find the value of  $y$ .

19. The values of  $x$  and  $y$  are found below. Write a justification for each step.

$$\frac{4}{x} = \frac{x}{4 + 5}$$

**Write a proportion.**

$$\frac{4}{y} = \frac{y}{5}$$

$$x^2 = 36$$

**Cross Products Property**

$$y^2 = 20$$

$$x = \sqrt{36}$$

**Take the positive square root.**

$$y = \sqrt{20}$$

$$x = 6$$

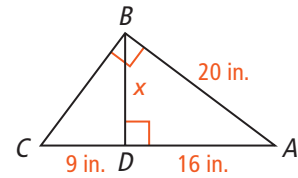
**Simplify.**

$$y = 2\sqrt{5}$$



## Problem 4 Finding a Distance

**Got It?** Points  $A$ ,  $B$ , and  $C$  are located so that  $AB = 20$  in., and  $\overline{AB} \perp \overline{BC}$ . Point  $D$  is located on  $\overline{AC}$  so that  $\overline{BD} \perp \overline{AC}$  and  $DC = 9$  in. You program a robot to move from  $A$  to  $D$  and to pick up a plastic bottle at  $D$ . From point  $D$ , the robot must turn right and move to point  $B$  to put the bottle in a recycling bin. How far does the robot travel from  $D$  to  $B$ ?



20. Place a  $\checkmark$  in the box if the statement is correct. Place an  $\times$  if it is incorrect.

- ☒ I know the length of the hypotenuse of  $\triangle ABC$ .
- ☒ I know the lengths of the segments of the hypotenuse of  $\triangle ABC$ .
- ☒ I know the length of the altitude of  $\triangle ABC$ .
- ☒ I can use Corollary 1 to solve the problem.

21. Find the length of  $\overline{BD}$ .

$$\begin{aligned}\frac{9}{x} &= \frac{x}{16} \\ x^2 &= 144 \\ x &= \sqrt{144} \\ x &= 12\end{aligned}$$

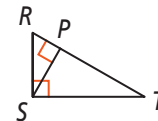
22. The robot travels **12** in. from  $D$  to  $B$ .



## Lesson Check • Do you UNDERSTAND?

**Vocabulary** Identify the following in  $\triangle RST$ .

- 23. The hypotenuse is  **$\overline{RT}$** .
- 24. The segments of the hypotenuse are  **$\overline{RP}$**  and  **$\overline{TP}$** .
- 25. The segment of the hypotenuse adjacent to leg  $\overline{ST}$  is  **$\overline{TP}$** .



## Math Success

Check off the vocabulary words that you understand.

- ☐ geometric mean      ☐ altitude      ☐ similarity

Rate how well you understand *similar right triangles*.

